NOTE ON PROFESSOR E. W. BROWN'S VERIFICATION OF THE CAPTURE OF SATELLITES.

By T. J. J. SEE.

In the *Monthly Notices* of the Royal Astronomical Society for March, 1911, Professor E. W. BROWN, of Yale University, has an important paper "On a New Family of Periodic Orbits in the Problem of Three Bodies." It includes an independent confirmation of the capture of satellites from among the asteroids transferred from beyond *Jupiter's* orbit to the region of the asteroid belt within, and is therefore important for our newer theories of cosmogony. We shall here give an outline of Professor BROWN's line of argument.

He remarks that in the so-called Trojan group—*Hector, Achilles, Petroclus,* and another as yet unnamed—there are four known asteroids which appear to oscillate about one or other of the vertices of the two equilateral triangles which have the line joining *Jupiter* and the Sun as a base. In these vertices asteroids of small mass would revolve in relative equilibrium, according to the particular solution of the problem of three bodies, discovered by LAGRANGE in the latter part of the eighteenth century; but LINDERS has remarked[1] that the heliocentric radius vector of one of these asteroids can apparently oscillate as far as 17° away from its equilibrium position. BROWN proceeds to inquire whether oscillations of this kind of still greater extent are possible, and if so, in what manner the orbits may be most conveniently obtained. He adopts for this inquiry the method of the problem of three bodies, with rotating axes, as treated by DARWIN,[2] which was made the basis also of my researches on the capture of satellites a little over two years ago.[3]

It is not permissible in this notice to go into the mathematical details of the reasoning, but it suffices to say that BROWN considers the stability of asteroids oscillating around the triangular points, and by certain criteria finds that the

[1] Arkiv för Mat., Ast. Och Fys., So. Vet. Ak., Stockholm, Bd. 4, No. 20.
[2] "Acta Mathematica," Vol. XXI, p. 103.
[3] "Researches on the Evolution of the Stellar System," Vol. II, 1910.

motions in the orbits about these triangular points are characterized by small velocity, and that any orbit cannot depart much from its zero velocity curve, and can only cross those members of the family $2\Omega - C' = 0$, for which $C' > C$. In other words, orbits near the zero velocity curves are essentially stable when the asteroid is of small mass.

Professor BROWN gives elaborate diagrams of the zero velocity curves for the region of *Jupiter's* orbit, with the constant of the Jacobian integral ranging from $C = 3.00286$ to $C = 3.04260$, corresponding, respectively, to the triangular points and the critical value where the Hill surface about *Jupiter* becomes closed. These curves were calculated and drawn by Professor BROWN'S assistant, Mr. H. B. HEDRICK, formerly of the Nautical Almanac Office, and are of course similar to the curves given by DARWIN in his celebrated memoir on "Periodic Orbits,"[1] except that in the present work of HEDRICK the curves are traced more in detail and the transition points brought out more clearly.

Professor BROWN investigated the stability of the orbits near these surfaces with considerable detail, adopting the method of HOUGH,[2] and shows that when the orbit passes through the equilibrium point, in opposition to *Jupiter,* there are two orbits, the ends of which closely approach the point, but the portions of these orbits in the neighborhood of the equilibrium point correspond to the unstable solutions which are obtained when we form the equations for small oscillations about the position of equilibrium. The equations of the orbits are found, and the directions of motion are shown to agree with those in the orbits; and it is proved that there is no discontinuity in the family of orbits while C passes through the critical value 3.04260. The values from $C = 3.04132$ to $C = 3.04260$ for the Jacobian integral give the zero velocity curves passing through the equilibrium points in the neighborhood of *Jupiter,* the opposition point being the one of widest range, and through which the periodic orbits will pass. BROWN shows that the orbits for values of C well beyond $C = 3.04260$ "consist (1) of an inner planetary orbit making complete revolutions round the Sun in the positive sense; (2) of an

[1] "Acta Mathematica," Vol. XXI.

[2] "Acta Mathematica," Vol. XXIV, p. 260.

outer planetary orbit making complete revolutions round the Sun in the negative sense relative to the moving axes; (3) of a satellite revolving round Jupiter in the positive sense." In other words, an asteroid passing through these equilibrium points with suitable velocity corresponding to the critical value of C, may pass under the control of *Jupiter* and become a satellite of that planet, as I pointed out in 1909.[1] And if the constant and velocity differ from the critical value by a small amount, changes due to the action of a resisting medium or the disturbance of a fourth body may give the critical values and enable the asteroid to be captured and become a satellite.

Professor BROWN concludes his paper with the following summary of the results as applied to the planetary system:

"Starting from infinitely small orbits round the triangular points, which expand mainly along the circle $r = 1$ until they join to form horseshoe orbits, the series progresses by the opposite ends of the horseshoe joining, so that the orbits separate into an outer planetary orbit and an inner hour-glass orbit, the latter afterwards separating into an inner planetary orbit and a satellite orbit. The final stage as C approaches infinity is an orbit of infinite radius and orbits of zero radius round the Sun and *Jupiter*.

"The classification of the family according to values of C is not, however, one which is natural in the consideration of the arrangement of the solar system. If we regard the orbits from the point of view of the mean distance from the Sun, they show a mode of transition, perhaps the most probable, for bodies moving in orbits superior to the planet to bodies in orbits inferior to the planet. It is seen that there are four possibilities. The superior planet may become an inferior planet, a satellite of *Jupiter*, or a planet oscillating about either of the two triangular equilibrium points, without any break in the continuity of the orbits."

This demonstration of the possibility of the capture of satellites confirms my conclusions of 1908 by an elaborate and critical inquiry, and it is impossible to overrate the significance of the results for the cosmogony of both the asteroids and the satellites. The transference of the asteroids within *Jupiter's* orbit by the pertubative action of that planet was concluded by

[1] *Astronomische Nachrichten*, 4341-42.

H. A. NEWTON as far back as 1894, and CALLANDREAU afterwards elaborated the theory of the capture and disintegration of comets.

Professor BROWN'S recent address to the American Association for the Advancement of Science[1] contained his first announcement that the Trojan group of asteroids "appear to show one, perhaps the main, stage of transition from bodies superior to the orbit of *Jupiter* to those inferior to that planet and possibly to those which have become his satellites. Their separate paths of motion are interesting to the mathematician, but even more so to the astronomer, since they appear to indicate a new set of periodic orbits in the problem of three bodies. The remarkable series of families of such orbits obtained by Sir GEORGE DARWIN has shown how far such an investigation may lead. . . . Theories as to the mode of formation of our solar system will, I believe, receive some assistance from these orbits of transition."

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PERSONAL EQUATION AND ITS VARIATION—DISCUSSION OF SOME CIRCUMSTANCES CAUSING RECORDED TIME OF A STAR'S TRANSIT TO DIFFER FROM TRUE TIME.[2]

By ELLIOTT SMITH.

The recorded time of transit of a star across a wire in the reticle of a transit instrument is rarely the true time. It may be obtained by applying a correction, commonly called personal equation, to the recorded time of transit.

An observer finds that this correction is not the same for all stars and that it varies with the conditions under which observations are made. It is different for stars of different magnitudes and is affected by the observer's position in making the observations. Variation of illumination of the field also produces a variation in this correction.

[1] *Science* for January 20, 1911, p. 93.

[2] Synopsis of thesis published in partial fulfillment of the requirements for the degree of doctor of philosophy in the University of Cincinnati.